A NEW BASIS FOR THE EFFICIENT SIMULATION OF SWITCH-MODE POWER CONVERTERS USING MULTIRATE PARTIAL DIFFERENTIAL EQUATIONS

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The simulation of switch-mode power converters with conventional time discretization is computationally expensive since very small time steps are needed to appropriately account for steep transients occurring inside the converter. An efficient simulation is obtained using Multirate Partial Differential Equations (MPDEs), which allow for a separation into components of different time scales. Solving the fast time scale with a Galerkin approach increases the size of the equation system. In this contribution a novel set of basis functions is presented allowing for a decoupling of the equation systems and thus an easy parallelization of the method.

I. Introduction

The MPDE approach using dedicated PWM basis functions achieves an efficient simulation scheme for problems with pulsed excitation and highly different time scales [1], e.g., power converters as the buck converter depicted in Fig. 1. Its solution consists of a slowly varying envelope and fast periodically varying ripples. Increasing the number of small time scale basis functions increases the size and fill-in of the arising block matrices substantially. This effect is diminished by a change of basis.

II. Multirate Partial Differential Equations

Let the linear differential equation describing the power converter be given as

$$\mathbf{A}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t)+\mathbf{B}\mathbf{x}(t)=\mathbf{c}(t), \qquad \mathbf{x}(0)=\mathbf{x}_0, \tag{1}$$

where $\mathbf{A}$, $\mathbf{B}$ are matrices, $\mathbf{x}(t)$ is the vector of N_s unknowns and $\mathbf{c}(t)$ the right-hand side. Furthermore $\mathbf{x}_0$ is the vector of initial values. Introducing an additional time scale leads to the corresponding MPDEs [2], [3]

$$\mathbf{A}\left(\frac{\partial\widehat{\mathbf{x}}}{\partial t_1}+\frac{\partial\widehat{\mathbf{x}}}{\partial t_2}\right)+\mathbf{B}\,\widehat{\mathbf{x}}(t_1,t_2)=\widehat{\mathbf{c}}(t_1,t_2)\,, \tag{2}$$

where $\widehat{\mathbf{x}}(t_1,t_2)$ is the unknown multivariate solution and $\widehat{\mathbf{c}}(t_1,t_2)$ the multivariate excitation. Choosing the multivariate excitation as such that $\widehat{\mathbf{c}}(t,t)=\mathbf{c}(t)$, the solution of (1) and (2) are related by $\widehat{\mathbf{x}}(t,t)=\mathbf{x}(t)$ [2], [3]. Thus, the solution of (1) can be calculated solving the MPDEs and extracting the solution along a diagonal through the computation domain.

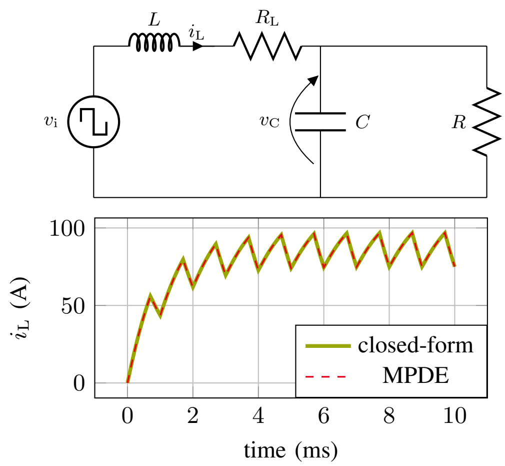

Fig. 1: Simplified circuit of the buck converter in continuous conduction mode (top) and solution calculated with the new basis functions compared with the closed-form solution (bottom).

To solve the MPDEs (2), a Galerkin approach and time discretization is applied [4]. The solution is expanded into periodic basis functions p_k depending on the fast time scale t_2 and coefficients $w_{j,k}$ depending on the slow time scale t_1

$$\widehat{x}_j^h(t_1,t_2):=\sum_{k=0}^{N_\mathrm{p}}p_k(\tau(t_2))\,w_{j,k}(t_1)\,, \tag{3}$$

where the periodicity of the basis functions is taken into account using the function $\tau=\frac{t_2}{T_\mathrm{s}}$ modulo 1. Applying the Galerkin approach with respect to t_2 and over one period of the excitation $[0,T_\mathrm{s}]$ leads to

$$\boldsymbol{\mathcal{A}}\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t_1}+\boldsymbol{\mathcal{B}}\,\mathbf{w}(t_1)\;=\;\boldsymbol{\mathcal{C}}(t_1)\,, \tag{4}$$

where, using the Kronecker product $\otimes$,

$$\boldsymbol{\mathcal{A}}=\boldsymbol{\mathcal{I}}\otimes\mathbf{A},\quad \boldsymbol{\mathcal{B}}=\boldsymbol{\mathcal{I}}\otimes\mathbf{B}+\boldsymbol{\mathcal{Q}}\otimes\mathbf{A}, \tag{5}$$

$$\boldsymbol{\mathcal{C}}(t_1)=\int_0^{T_\mathrm{s}}\bar{\mathbf{p}}(\tau(t_2))\otimes\widehat{\mathbf{c}}(t_1,t_2)\mathrm{d}t_2\,,\ \text{and} \tag{6}$$

$$\boldsymbol{\mathcal{I}}=T_\mathrm{s}\int_0^1\mathbf{p}(\tau)\,\mathbf{p}^\star(\tau)\,\mathrm{d}\tau,\ \ \boldsymbol{\mathcal{Q}}=-\int_0^1\frac{\mathrm{d}\mathbf{p}(\tau)}{\mathrm{d}\tau}\,\mathbf{p}^\star(\tau)\,\mathrm{d}\tau\,. \tag{7}$$

The $\star$ denotes the complex conjugate transposed.

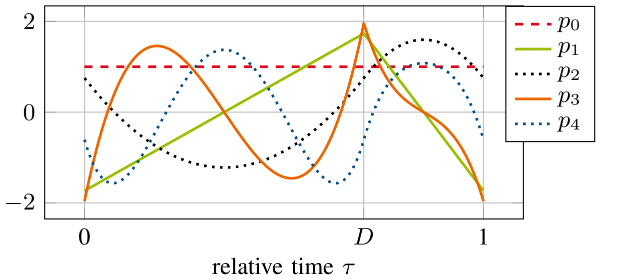

Fig. 2: Original PWM basis functions $p_k(\tau)$, $k \in \{0, 1, 2, 3, 4\}$.

III. PWM BASIS FUNCTIONS

The PWM basis functions developed in [5] are built up starting from the zero-th constant basis function $p_0(\tau) = 1$ and the piecewise linear basis function

$$p_1(\tau) \;=\; \begin{cases} \sqrt{3}\,\frac{2\tau - D}{D} & \text{if} \quad 0 \le \tau \le D \\ \sqrt{3}\,\frac{1+D-2\tau}{1-D} & \text{if} \quad D \le \tau \le 1 \end{cases}, \qquad (8)$$

which includes the duty cycle D of the excitation by construction. The higher-order basis functions $p_k(\tau)$, $2 \le k \le N_\mathrm{p}$ are recursively obtained by integrating the basis functions of lower order $p_{k-1}(\tau)$ and orthonormalizing them. The so-generated basis functions are depicted in Fig. 2.

For the PWM basis functions, the matrices $\boldsymbol{\mathcal{I}}$ and $\boldsymbol{\mathcal{Q}}$ from (7) are given by the identity matrix and a full matrix with around 25 % of non-zero entries, respectively. The degrees of freedom in the equation system (4) cannot be decoupled.

IV. TRANSFORMED PWM BASIS FUNCTIONS

To enable an easy parallelization of the method, the equations (4) need to be decoupled, for example by diagonalizing $\boldsymbol{\mathcal{Q}}$, i.e., a basis transformation. We define new basis functions being linear combinations of the PWM basis functions, i.e.,

$$g_k(\tau) = \sum_{l=0}^{N_\mathrm{p}} v_{k,l}\, p_l(\tau),$$

where $v_{k,l}$ are unknown coefficients, and enforce $g_k(\tau)$ to be eigenfunctions of the time derivative operator

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\, g_k(\tau) = \lambda_k\, g_k(\tau) \qquad (9)$$

in a weak sense. This is achieved by a Galerkin approach, i.e.,

$$-\int_0^1 g_k(\tau)\frac{\mathrm{d}p_m(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau \;=\lambda_k \int_0^1 g_k(\tau)p_m(\tau)\mathrm{d}\tau. \qquad (10)$$

which, after inserting the expansion of the basis functions, can be written as

$$T_\mathrm{s}\boldsymbol{\mathcal{Q}}\mathbf{v}_k = \lambda_k \mathbf{v}_k \boldsymbol{\mathcal{I}}. \qquad (11)$$

Since $\boldsymbol{\mathcal{I}}$ is the identity matrix (thanks to the orthonormality of the PWM basis functions), the λ_k and $\mathbf{v}_k$ are the eigenvalues and eigenvectors of the matrix $\boldsymbol{\mathcal{Q}}$, respectively. Furthermore since $\boldsymbol{\mathcal{Q}}$ is real skew symmetric and therefore a normal matrix, i.e., $\boldsymbol{\mathcal{Q}}^\top\boldsymbol{\mathcal{Q}} = \boldsymbol{\mathcal{Q}}\boldsymbol{\mathcal{Q}}^\top$, the eigenvectors $\mathbf{v}_k$ are orthonormal. The new basis functions (complex-valued) are depicted in Fig. 3 for $N_\mathrm{p} = 4$.

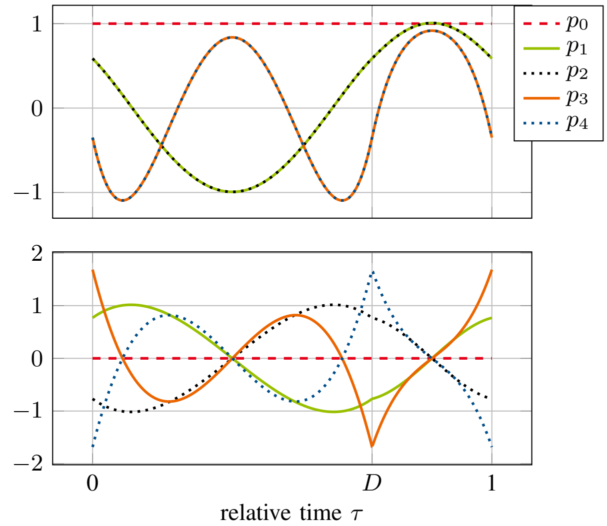

Fig. 3: Transformed PWM basis functions $g_k(\tau)$, $k \in \{0, 1, 2, 3, 4\}$, i.e., $N_\mathrm{p} = 4$. (top) real part. (bottom) imaginary part.

Inserting the new basis functions instead of the PWM basis functions into (6),(7) leads, using the orthonormality of the eigenvectors, to the block-diagonal problem

$$\widetilde{\boldsymbol{\mathcal{A}}}\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}t_1} + \widetilde{\boldsymbol{\mathcal{B}}}\,\mathbf{w}(t_1) \;=\; \widetilde{\boldsymbol{\mathcal{C}}}(t_1)\,, \qquad (12)$$

where, using the Kronecker product $\otimes$,

$$\widetilde{\boldsymbol{\mathcal{A}}} = \boldsymbol{\mathcal{I}} \otimes \mathbf{A}, \quad \widetilde{\boldsymbol{\mathcal{B}}} = \boldsymbol{\mathcal{I}} \otimes \mathbf{B} + \boldsymbol{\lambda} \otimes \mathbf{A}, \qquad (13)$$

$$\widetilde{\boldsymbol{\mathcal{C}}}(t_1) = \int_0^{T_\mathrm{s}} \bar{\mathbf{g}}(\tau(t_2)) \otimes \hat{\mathbf{c}}(t_1, t_2)\mathrm{d}t_2\,, \qquad (14)$$

and $\boldsymbol{\lambda}$ is a diagonal matrix with diagonal entries $\lambda_0, \lambda_1, \ldots, \lambda_{N_\mathrm{p}}$. Thus the resulting matrices in (13) are block-diagonal and the degrees of freedom can be decoupled. Fig. 1 shows the analytic solution of the buck converter compared to the solution calculated with the newly developed basis functions.

V. CONCLUSION

The new eigenfunctions turn the MPDEs into a complex-valued block-diagonal system and therefore allow for efficient parallelization.

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